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NUMERICAL SIMULATION OF THE PHENOMENA DUE TO THE PASSING-BY OF TWO BODIES USING THE UNSTEADY BOUNDARY ELEMENT METHOD

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SUMMARY

In this paper a numerical analysis was made to investigate the aerodynamic forces surrounding two bodies in relative motion in a fluid at rest in three dimensions. The unsteady boundary element method was employed in the numerical calculations. This method is very convenient for obtaining an approximate expression of the velocity potential, especially for practical use. The passing-by of two spheres in an incompressible perfect fluid which extends to infinity is treated by the present method. The resultant pressure coefficients on two spheres passing each other in opposite directions are calculated and discussed numerically. Numerical examples are presented to show the validity of the present method. The method is also applied to the calculation of the passing-by of two trains in an open area in order to investigate its applicability.

KEY WORDS: boundary element method; unsteady aerodynamic force; relative motion

1. INTRODUCTION

The steady flow of a perfect fluid past an obstacle has been investigated by many authors, but there have been few works on the unsteady flow due to the motion of several bodies. The investigation of such a flow became necessary in connection with the high speed of new vehicles.

At high speed, aerodynamic problems occur particularly when other vehicles pass by (Figure 1). For example, one vehicle experiences a sudden inverse pressure due to the passage of another vehicle. The flow field when vehicles pass by should be investigated in order to develop effective vehicle configurations for the future.

The first study of this flow field was conducted by Kawaguchi in 1963.[1] The flow field around one train passing another was analytically studied and the effect of the distance between the two trains on the pressure was investigated. Some features of the flow make the study difficult. The boundaries of

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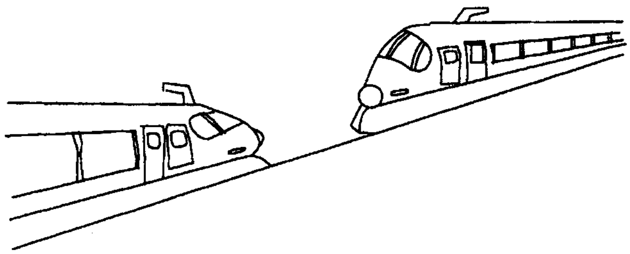

Figure 1. Passing-by of trains

the flow field change with time. Experimental studies of this configuration would require great skill because of the relative motion of the boundaries. It is quite difficult to obtain flow data around a moving body. Analytical studies are also difficult owing to the moving boundary configuration and its unsteadiness.

In this paper we shall investigate the unsteady flow of a perfect fluid caused by the motion of two spheres in three dimensions which pass by each other, in order to estimate the pressure variation when two high-speed vehicles pass each other. To this end, computational fluid dynamics (CFD) is used for the investigation of the flow field. CFD can be a good tool for the investigation of flow fields to which experimental studies are hard to apply. However, the features mentioned above create difficulties also in CFD. The unsteady boundary element method is used to remove the difficulties in this research. There is no need to regenerate the panel at each time step unless the moving bodies change their shape. This is a great advantage of the unsteady boundary element method and it shortens the turnaround time in parametric studies. Further description of the numerical algorithm is presented in the following section. A series of phenomena are simulated and compared with analytical data.

2. FLOW AROUND TWO MOVING SPHERES

Kawaguchi has analytically treated the problem of two spheres passing by each other. In order to compare with our numerical result later, the outline of the analysis is described simply in the following (Figure 2).

Although the flow of a perfect fluid around a sphere of radius a which moves parallel to the x-axis at velocity U can be exactly expressed by placing the doublet of strength $Ua^3/2$, whose axis is parallel to the x-axis, at the centre of the sphere, the exact solution cannot be obtained by merely placing two doublets at two centres for the flow around two moving spheres (first approximation). To get a higher approximation for the flow around two moving spheres, it is necessary to add a flow due to the images of doublets with respect to spheres (second approximation), then to add a flow due to the images of images and so on.

3. THEORY

We shall consider the case where two spheres C_1 and C_2 of radius a (their centres being A_1 and A_2 respectively) pass by each other at velocities U and V parallel to the x-axis respectively. The instant when the two spheres come closest to each other is chosen as the origin of time ($t = 0$) and the middle point of A_1A_2 at $t = 0$ is taken as the origin of the co-ordinates, O. Let $2k$ be the shortest distance between the centres of the spheres at $t = 0$ (Figure 3).

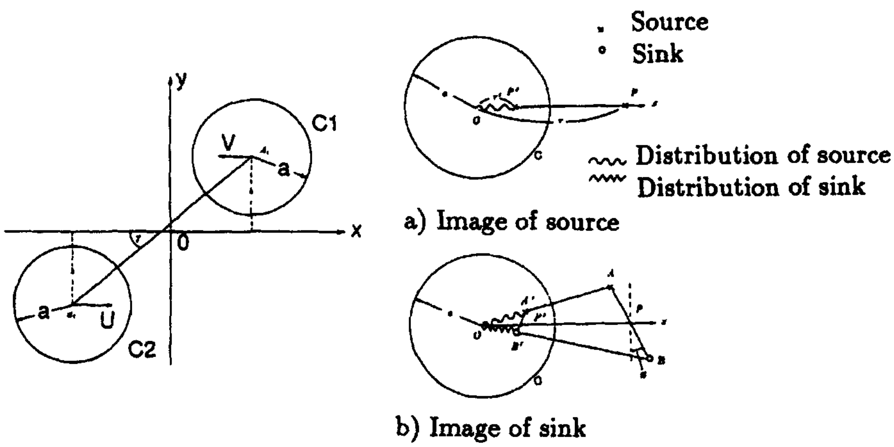

Figure 2. Three-dimensional model of two spheres passing by

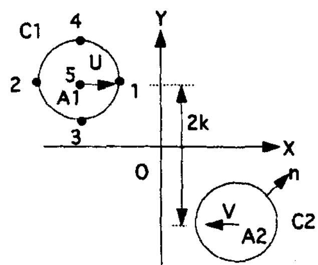

Figure 3. Modelling of two spheres in relative motion

The problem of passing-by spheres is solved as an initial value problem. At each time interval a boundary value problem is solved for the velocity potential outside the bodies.

If we consider three-dimensional bodies passing by at velocity u_w, assuming the flow field around the bodies to be incompressible, inviscid and irrotational, a velocity potential ϕ exists which must obey Laplace's equation. The boundary condition is given on the body by

$$\frac{\partial \phi}{\partial n} = u_w \cdot n. \tag{1}$$

In order to determine ϕ at the body surface, Green's second identity is used. A Fredholm integral equation of the second kind is obtained for the velocity potential on the body surface:

$$\phi(P) - \frac{1}{2\pi}\int_S \phi(Q)\frac{\partial}{\partial n(Q)}\frac{1}{r(P,Q)}\,\mathrm{d}S = -\frac{1}{2\pi}\int_S \frac{\partial \phi(Q)}{\partial n(Q)}\frac{1}{r(P,Q)}\,\mathrm{d}S. \tag{2}$$

Here S is the surface of the body and $r(P, Q)$ is the distance between P and Q, where P and Q are two arbitrary points on S.

Following Morino's method,[2] the integral equation is approximated in a system of linear algebraic equations using the finite element technique combined with the collocation method:

$$\sum_k (\delta_{hk} - C_{hk})\phi_k = -\sum_k B_{hk}\left(\frac{\partial \phi}{\partial n}\right)_k. \tag{3}$$

Here δ_{hk} is the Kronecker symbol, while C_{hk} and B_{hk} are aerodynamic influence coefficients defined by

$$C_{hk} = \left(\frac{1}{2\pi}\int_S \frac{\partial}{\partial n}\frac{1}{r}\,\mathrm{d}S\right)_{P=P_k}, \qquad B_{hk} = \left(\frac{1}{2\pi}\int_S \frac{1}{r}\,\mathrm{d}S\right)_{P=P_k}. \tag{4}$$

These integrals are analytically evaluated in detail without using the numerical integral method. Equation (3) can be solved numerically to yield the value of unknown ϕ_k.

The x-, y- and z-components of velocity at any point are easily given by differentiating (3):

$$2E_h v_h = -\sum_k \nabla_* B_{hk}\left(\frac{\partial \phi}{\partial n}\right)_k + \sum_k \nabla_* C_{hk}\phi_k, \tag{5}$$

with

$$\nabla_* C_{hk} = \left(\frac{1}{2\pi}\int_S \nabla_* \frac{\partial}{\partial n}\frac{1}{r}\,\mathrm{d}S\right)_{P=P_k}, \qquad \nabla_* B_{hk} = \left(\frac{-1}{2\pi}\int_S \nabla_* \frac{1}{r}\,\mathrm{d}S\right)_{P=P_k}. \tag{6}$$

The surface elements of the body are approximated by portions of a hyperboloidal paraboloid and the coefficients $\nabla_* C_{hk}$ and $\nabla_* B_{hk}$ can be evaluated analytically[3] as the coefficients C_{hk} and B_{hk}.

The pressure on the surface can be obtained from an extended Bernoulli formula

$$\frac{\partial \phi_h}{\partial t} + \tfrac{1}{2}q_h^2 + p_h = f(t). \tag{7}$$

Through the first term on the left-hand side we include unsteadiness. Evaluation of the first term on the left-hand side is not easy. There are many methods proposed by various researchers.[4,5] In this study we evaluate the term as

$$2E_h \frac{\partial \phi_h}{\partial t} = \sum_k \left[-\nabla_* C_{hk} \cdot u_k \phi_k + \nabla_* B_{hk} \cdot u_k \left(\frac{\partial \phi}{\partial n}\right)_k\right]. \tag{8}$$

Using this equation, we can get accurate results and shorten the calculation time.

4. RESULTS

The numerical model is shown in Figure 4. The discretization used here is $(N_C = 30) \times (N_S = 16)$ so that the total number of panels of one sphere is 480. The non-dimensional time interval is $Ut/a = 0{\cdot}1$, where U is the larger velocity of the two spheres. One sphere should be placed far away from the other sphere at the beginning. However, it would require an enormous computational time for the sphere to travel over a long distance towards the other sphere and much of the computer time would be spent on the region that is outside our interest. Therefore we put the non-dimensional calculation time at -4 to 4 so that the total number of time steps is 80.

Numerical calculations have been carried out for the following cases:

(1) $U = 1$, $V = -1$, $k = 1{\cdot}25a$
(2) $U = 1$, $V = 0$, $k = 1{\cdot}25a$
(3) $U = 0$, $V = -1$, $k = 1{\cdot}25a$.

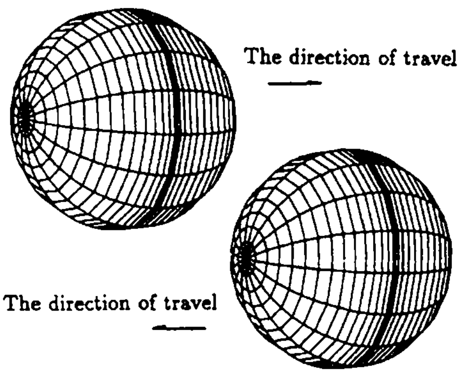

Figure 4. Numerical model of two spheres

These cases are shown in order to see the effect of changing the velocities of the two spheres. Case (1) is when both spheres are moving and cases (2) and (3) are when one of the spheres is at rest and the other is moving. Cases (2) and (3) were studied to clarify the phenomena when a train stops at a station and another train passes.

The velocity components at various points on the sphere C_1 in these cases are given in Figure 5. From these velocities we can calculate the pressure distribution. The pressure variations on the sphere C_1 in the above cases are given in Figure 6.

The pressure variations shown in Figure 6 are also compared with the numerical results and analytical solutions. We can see good agreement with the analytical values.

We can see the behaviour of the time sequences of the pressure variation on the sphere in the figures as follows.

(a) As expected, the pressure on the meeting side (point 3) is larger than that on the outer side (point 4).
(b) When one of the two spheres is at rest, the pressure change on the resting sphere is larger than that on the moving one and the pressure variation is similar to that when both are moving.
(c) The pressure around the stagnation point shows a weak maximum and then falls to a strong minimum just before the other sphere comes by.

It is not easy to calculate the side force acting on the sphere by Kawaguchi's method, but we can obtain it easily using our method. The side force acting on the sphere is shown in Figure 7. As expected, it is noted that when both spheres are moving, i.e. case (1), the peak-to-peak value of the side force is largest. When one of the two spheres is at rest, the change in the side force on the resting sphere is larger than that on the moving one.

5. APPLICATION

We applied the present method to the calculation of the passing-by of two trains in an open area in order to investigate its applicability. The field test result[6] is given in Figure 8. This figure shows the pressure variation on the meeting side of the middle car. The two trains pass by on the open ground at velocities of 260 and 210 km h^{-1} respectively.

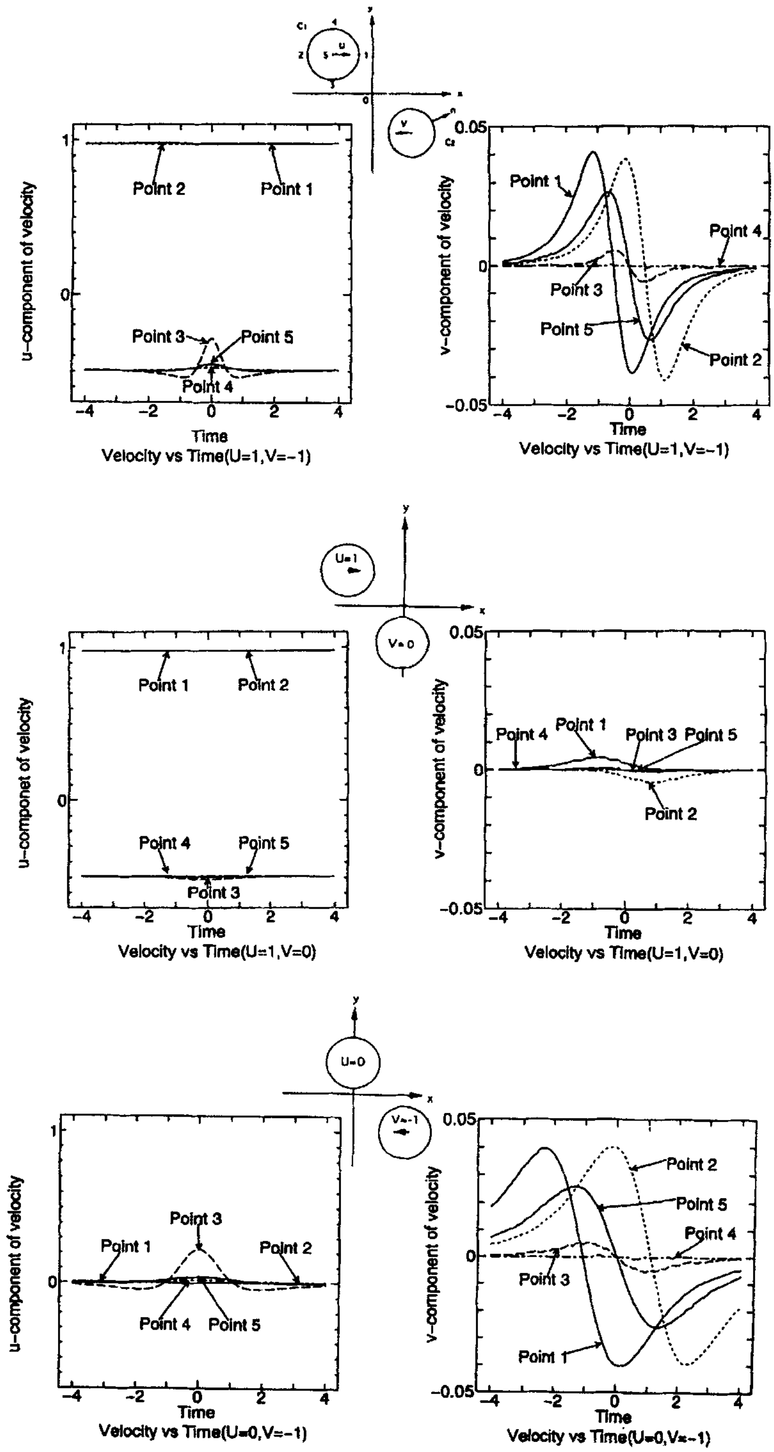

Figure 5. Time sequences of velocity

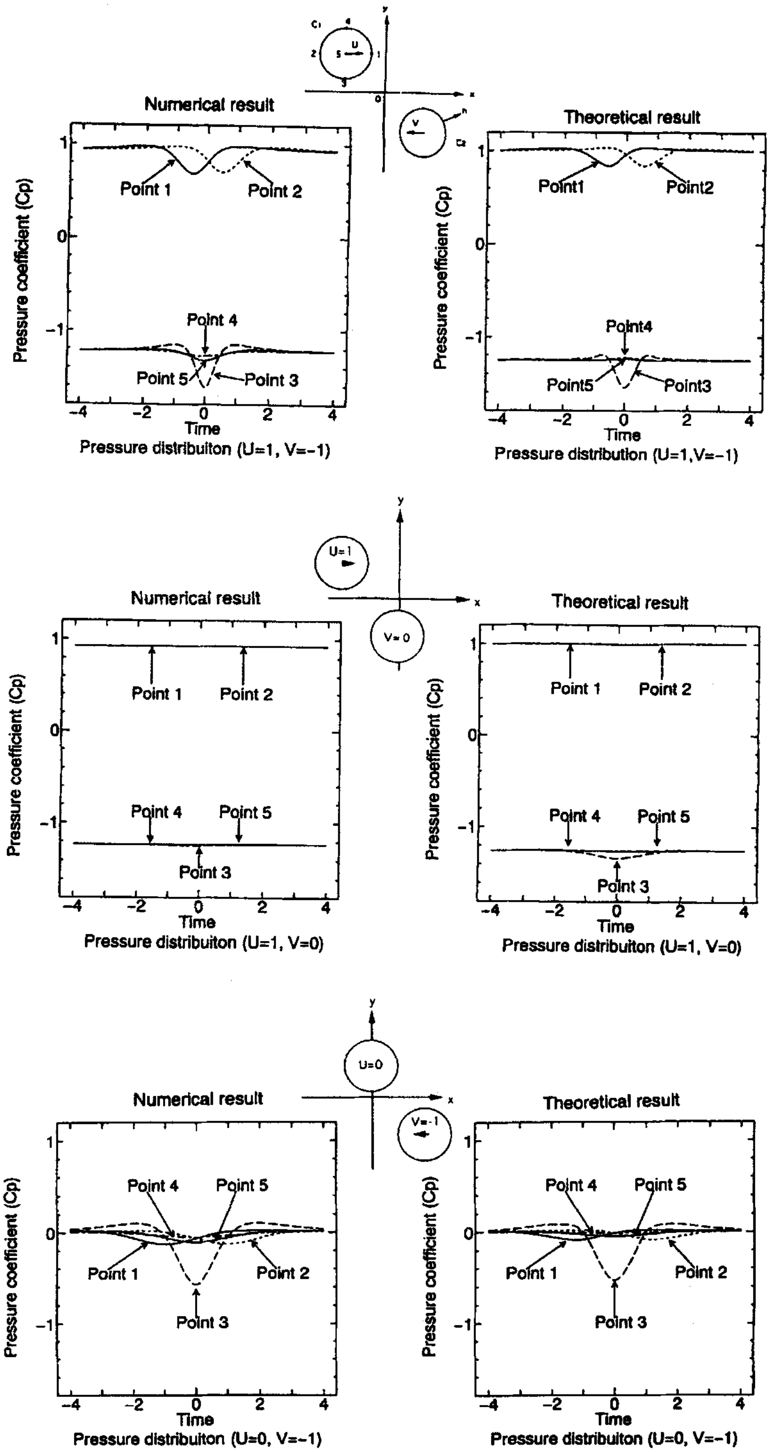

Figure 6. Time sequences of pressure

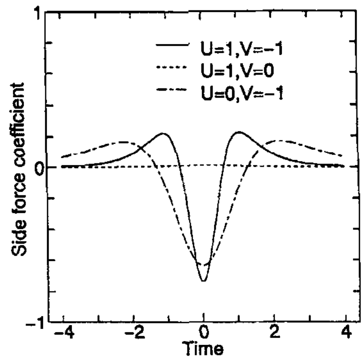

Figure 7. Time sequences of side force acting on sphere

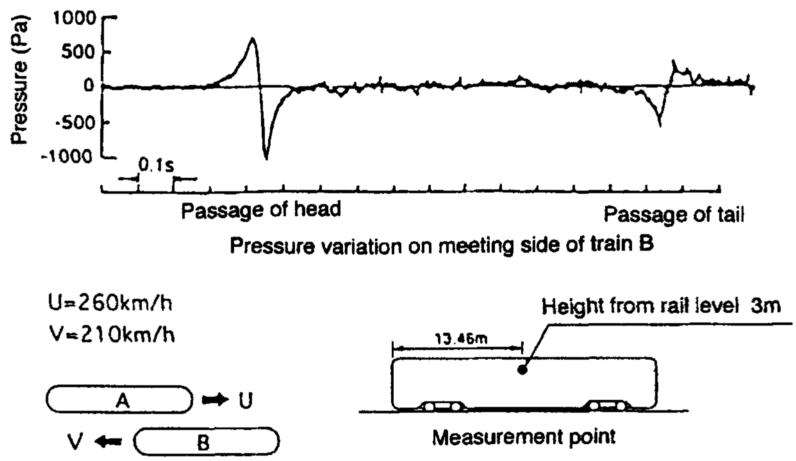

Figure 8. Field test result

We see in the figure that the pressure rises first, followed immediately by a sudden drop at the moment of the passage of the head of the opposite train. In addition to these pressure variations, there comes a negative pressure peak at the moment of the passage of the tail of the opposite train, but this is smaller than the former owing to the viscosity.

The numerical model is shown in Figure 9. The discretization used here is $(N_C = 85) \times (N_S = 12)$ so that the total number of panels of one train is 1020.

The pressure variation of the middle car is shown in Figure 10. The thicker line indicates the pressure variation on the meeting side. The thinner line indicates the pressure variation on the outer side. We can see the same phenomena of pressure variation as in the field test result. Because of potential theory, the peak-to-peak value at the moment of the passage of the tail is same as that at the moment of the passage of the head. Comparisons between numerical results and field test results show qualitatively good agreement.

We can also see that the pressure variation of the passing-by train can be predicted reasonably well by the present method.

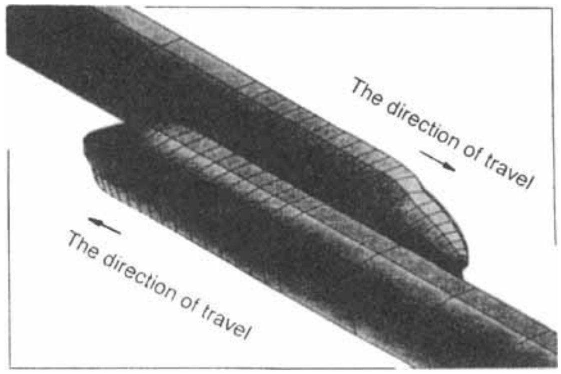

Figure 9. Numerical model of two trains

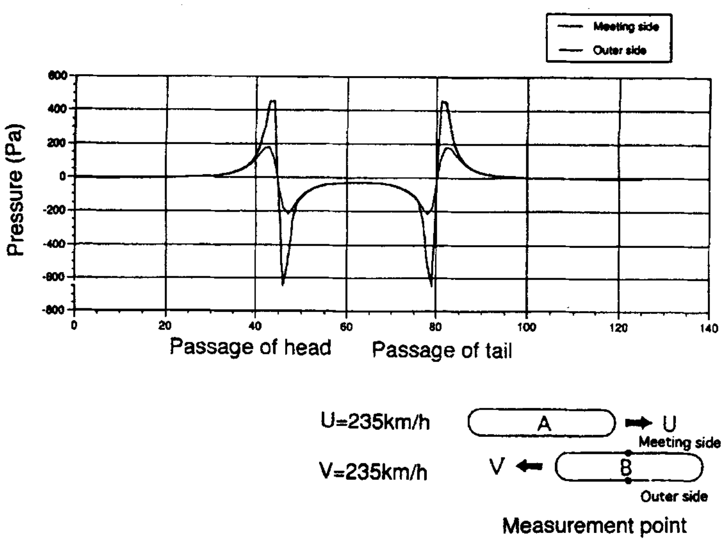

Figure 10. Time sequences of pressure

6. CONCLUSIONS

Three-dimensional inviscid flows induced by spheres passing by each other have been computationally simulated using the unsteady boundary element method. Three flow configurations were investigated.

From the results shown above, the following conclusions were obtained.

(1) The unsteady boundary element method is effective for calculating the flow around two moving spheres.
(2) The calculated results agree with analytical data from the image method.

We also applied the present method to the calculation of two trains passing by in an open area. The results indicate that the numerical algorithm is sufficiently adaptable to the modelling of passing-by bodies.

There is no need to regenerate the panel at each time step by the unsteady boundary element method. This is a great advantage of the unsteady boundary element method and it shortens the turnaround time in practical work.

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